A NEW LOGICAL MACHINE.

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During the year 1881 I constructed a logical machine somewhat similar to the well-known machine of Prof. Jevons, and printed logical diagrams for problems involving as many as ten terms.* This earlier instrument and the logical diagrams formed the basis of the machine illustrated on the accompanying plate. The new machine was constructed in Princeton during the winter of 1881–82, by my friend Prof. C. G. Rockwood, Jr., whose mechanical skill and untiring patience gave me invaluable assistance. The machine was made from the wood of a red-cedar post, which once formed part of the enclosure of Princeton's oldest homestead. It measures 32 cm. high by 21 cm. wide and 15 cm. deep. Like the instrument of Prof. Jevons, and that of Prof. Venn, it is constructed for problems involving only four terms, but more readily than either of those instruments admits of being extended for problems involving a larger number of terms.

The face of the machine (Fig. I.) presents to view sixteen small pointers representing the sixteen logical combinations of the symbols A, B, C, and D, with their negatives a, b, c, d. These combinations are so arranged that all the A combinations are found in the two vertical columns to our left, the a combinations in the two vertical columns to the right. The A combinations are subdivided vertically into the B and b combinations, and the a series in like manner. Both are also subdivided horizontally into the C and c combinations, and each of these again into the D and d combinations. Thus the uppermost pointer to the left represents the combination $A\ B\ C\ D$, the horizontally adjoining one $A\ b\ C\ D$, the next $a\ B\ C\ D$, and so on until we reach the lowermost to the right, which is $a\ b\ c\ d$. Below the pointers may be seen the two operation keys marked *1* and *0*, and the four positive and four negative letter keys under their respective symbols.

* Philos. Mag., October, 1881, pp. 266–270.

The *1* key may be called the *restoration key*. It prepares the machine for a new problem, by raising all the pointers to the horizontal position. The *0* key may be called the *destruction key*, as when pressed down it allows the pointers to fall. The function of a *letter key* is to sustain in the horizontal position the pointers representing the corresponding negative combinations. Thus, if the *A* key is pressed, the *a* pointers are sustained; if the *b* key is pressed, the *B* pointers are sustained; and similarly for all the remaining letter keys.

Turning now to Fig. II., the mechanism by which this is effected will be easily seen. On the inner face of the machine, and corresponding to the pointers, are sixteen drops, some of which are pictured in the horizontal position, and some as fallen. The two operation keys move rectangular frameworks, each consisting of four vertical brass rods fastened together, and carrying pins which reach all of the sixteen drops. The framework of the *0* key moves on the inner, that of the *1* key on the outer side of the drops, both horizontally. If we wish to bring all the pointers to the horizontal position and hold them there, both operation keys are pressed down. The *1* key raises all the drops. Releasing first the *0* key, its framework is drawn back by a fixed spring, and by means of its pins holds the drops in position; on releasing the *1* key, its framework is drawn back so as not to interfere with subsequent operations. Each letter key operates two vertical or two horizontal rods, free to revolve on their axes. In each rod are set four pins, conveniently bent, so that when the rods are partially revolved, by pressure of the letter keys, the pins are made to sustain the drops in the horizontal position without raising those which have fallen. The rods return to their original position, by means of small spiral springs, as soon as the pressure of the letter keys is released. The desired motion is communicated to the rods by means of cat-gut strings, there being two such strings from each letter key. Thus the *A* key operates the two vertical rods which are to hold the *a* drops; the *b* key the rods for holding the *B* drops; and the other letter keys in like manner. By this device it will be seen that, if we should press the *A* key (this holds the *a* combinations), and then the *0* key, only the *A* pointers will fall; or, if both the *A* and *B* keys are depressed (this holds the *a* and *b* combinations), and then the *0* key, only the *A B* pointers will fall; and similarly for the other combinations.

To utilize the instrument for the solution of logical problems, we first raise all the pointers to the horizontal position. This will indicate the state of a logical universe of four terms before the introduction of premises. Now, since the establishment of any combination

means the negation of some other, we may express our premises in negative form. Thus, in general, $A \prec B$ (every A is B) may be expressed $A\ b \prec 0$ (A's which are b do not exist). This we express upon our machine by pressing down the letter keys A and b, and then the destruction key. The falling of the $A\ b$ pointers indicates exactly the change effected in the logical universe by the introduction of the premise $A \prec B$. We may then continue to impress as many premises as we please, until all the pointers have fallen. The following formulæ* will suffice to illustrate the manner in which premises may be reduced from the positive to the negative form.

	Positive Form.	Negative Form.
(1.)	$A \prec B$	$A\ b \prec 0.$
(2.)	$A + B \prec C$	$\begin{cases} A\ c \prec 0. \\ B\ c \prec 0. \end{cases}$
(3.)	$A \prec B + C$	$A\ b\ c \prec 0.$
(4.)	$A \prec b + C\ D$	$\begin{cases} A\ B\ c \prec 0. \\ A\ B\ d \prec 0. \end{cases}$
(5.)	$A\ B \prec C$	$A\ B\ c \prec 0.$
(6.)	$A \prec B\ C$	$\begin{cases} A\ b \prec 0. \\ A\ c \prec 0. \end{cases}$
(7.)	$A\ (B + C) \prec D$ . . .	$\begin{cases} A\ B\ d \prec 0. \\ A\ C\ d \prec 0. \end{cases}$
(8.)	$A \prec B\ (C + D)$ . . .	$\begin{cases} A\ b \prec 0. \\ A\ c\ d \prec 0. \end{cases}$

Having expressed our premises upon the machine, and their effect being recorded by the pointers, it only remains for us to read off the conclusion. The entire conclusion is represented by the fallen pointers, and might be expressed as their joint sum; or it may be viewed as the logical sum of the combinations represented by the horizontal pointers. Thus, the premises of Barbara, $A \prec B$ and $B \prec C$, give as the entire conclusion read negatively,

$$\left.\begin{matrix} A\ b\ C \\ A\ b\ c \\ A\ B\ c \\ a\ B\ c \end{matrix}\right\} \prec 0.$$

* The sign $\prec$ is that used by Mr. C. S. Peirce for the general sign of inference. $A \prec B$ means, if A, then B. Viewed in the light of class extension, it means the class A *is included in* the class B. The sign of addition is here used in the non-exclusive sense; thus, $A + B$ means either A or B, or both. The expression $A\ B$ means, when designating a class, the individuals which belong to both classes A and B; when designating a quality, the combination of the qualities A and B.

Read positively, the conclusion is,

$$A\,B\,C + a\,B\,C + a\,b\,C + a\,b\,c = 1\,;$$

or, more briefly,

$$B\,C + a\,b = 1.$$

Ordinarily, the conclusion called for is part only of the total conclusion. Thus, syllogism with the above premises asks for a conclusion involving only A and C. An inspection of the dial-plate will show us the conclusion $A \prec C$, and also other conclusions involving relations between other terms than A and C; thus,

$$c\,(A + B) \prec 0\,;\; b \prec a + c, \text{ etc.}$$

Nor is it necessary that our conclusions should be limited to relations between terms given in the premises, as may be seen in the solution of the following problems.

Problem I.

Let us suppose that there are four girls at school, Anna, Bertha, Cora, and Dora, and that some one had observed that

(1.) Whenever either Anna or Bertha (or both) remained at home, Cora was at home; and

(2.) When Bertha was out, Anna was out; and

(3.) Whenever Cora was at home, Anna was at home.

What information is here conveyed concerning Dora?

Indicating by the capital letters the fact of *remaining at home*, and by the small letters that of *going out*, our premises are

$$A + B \prec C = \left.\begin{matrix} A\,c \\ B\,c \end{matrix}\right\} \prec 0$$
$$b \prec a \qquad = b\,A \prec 0$$
$$C \prec A \qquad = C\,a \prec 0$$

and, impressing them upon the machine, there will result the state of things indicated by Fig. 1. From this we may read off the conclusion,

$$D \prec A\,B\,C + a\,b\,c.$$
$$d \prec A\,B\,C + a\,b\,c.$$

Or, if Dora remain at home, her three sisters will be all at home or all out; and the same will be true if Dora goes out.

Problem II.

If $A = B$ and $B = C$, what may be said of D?
Impressing upon the machine our premises,

$$(A = B) = \left.\begin{matrix} A \prec B \\ B \prec A \end{matrix}\right\} = \begin{matrix} A\,b \prec 0 \\ B\,a \prec 0 \end{matrix}$$

$$(B = C) = \left.\begin{matrix} B \prec C \\ C \prec B \end{matrix}\right\} = \begin{matrix} B\,c \prec 0 \\ C\,b \prec 0 \end{matrix}$$

the same state of the logical universe is produced as by the premises of the preceding problem. Hence,

$$D \prec A\,B\,C + a\,b\,c.$$

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