LOGARITHMIC-TRIGONOMETRICAL TABLES

WITH EIGHT DECIMAL PLACES

CONTAINING

THE LOGARITHMS OF ALL NUMBERS FROM 1 TO 200000 AND THE LOGARITHMS OF THE TRIGONOMETRICAL FUNCTIONS FOR EVERY SEXAGESIMAL SECOND OF THE QUADRANT

UNDER THE PATRONAGE OF THE ROYAL PRUSSIAN ACADEMY OF SCIENCES OF BERLIN AND THE IMPERIAL ACADEMY OF SCIENCES OF VIENNA

NEWLY CALCULATED AND PUBLISHED BY

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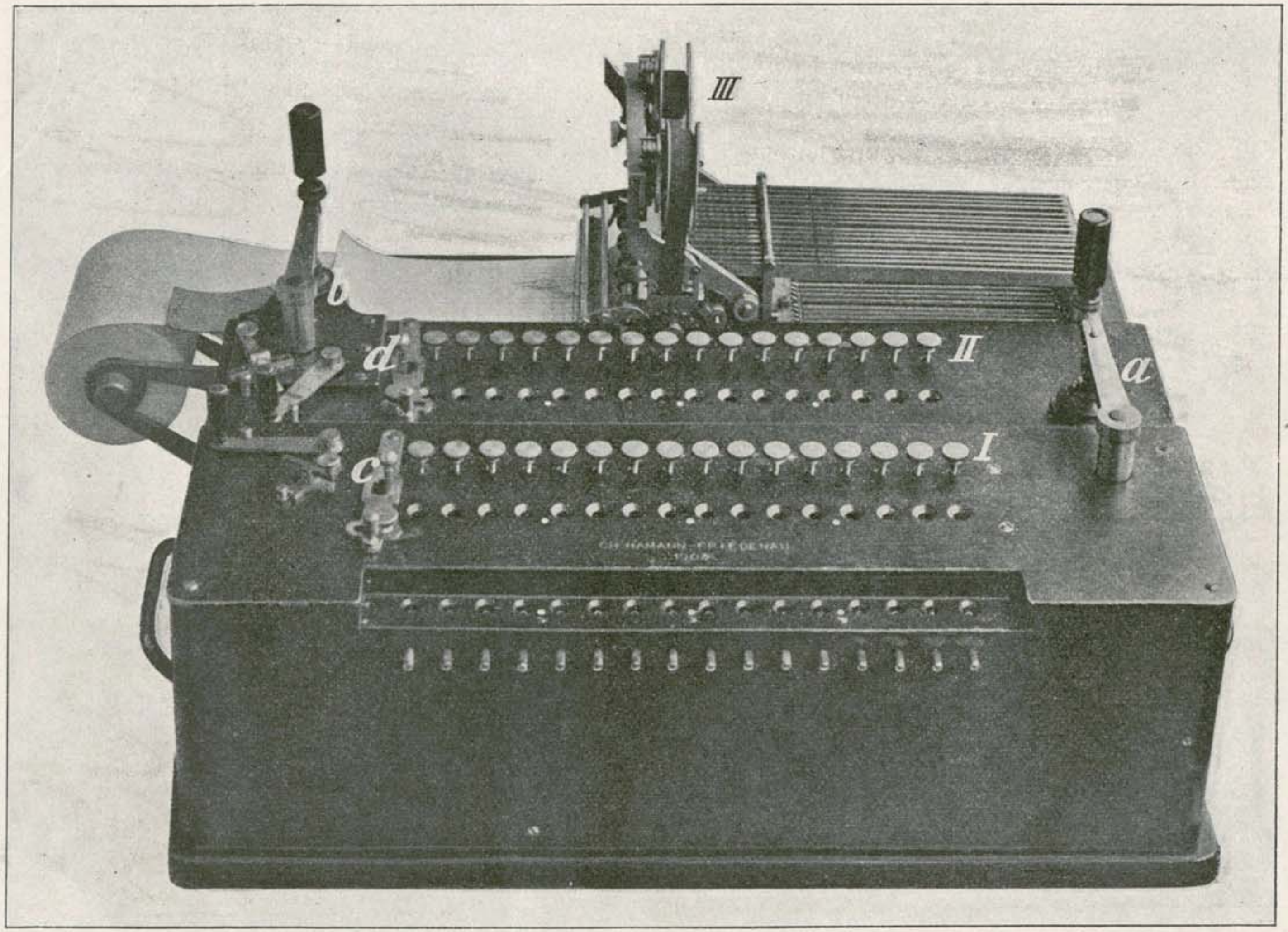
FIRST VOLUME

TABLE OF LOGARITHMS TO EIGHT PLACES OF ALL NUMBERS FROM 1 TO 200000

LEIPZIG

PUBLISHED BY WILHELM ENGELMANN

1910



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PREFACE.

The need for logarithm tables to eight places, a need which we hope to satisfy with this work, first made itself felt in connection with astronomy and geodesy, for the tables to seven places in common use were no longer sufficiently accurate for the greater degree of exactitude now required in observations.

But in statistics, too, in insurance tables, in technical calculations, in finance, etc., the cases became more and more numerous where the calculations necessary imperiously demanded a higher degree of exactness than was obtainable with the seven place tables. The one alternative open was to turn to the only available tables, those to 10 places, the Thesaurus of Vega, an alternative one adopted very unwillingly, as it far exceeded the degree of accuracy required, and consequently involved loss of time. It, moreover, led to very troublesome interpolations with second differences, since the interval of the above mentioned tables is too great a one.

The first real remedy was found in the »Table de logarithmes de Service géographique de l'armée« (Paris 1891), which, founded upon entirely independent calculation, presented the 8 place tables of logarithms in an excellently printed volume, the whole well-arranged and of the utmost reliability. The desire to produce a monumental work has, however, led to the adoption of too large a type and to a somewhat clumsy format, — things that, at least, prove inconvenient if the tables have to be frequently consulted. In some places, too, the interval is still an overlarge one and, last of all, the decimal division of the quadrant is used as a basis, a fact which makes the use of this work if not impossible, at least unpractical for astronomers and partly, too, for geodesy and other arithmetical sciences.

This being the case we, in 1904, conceived the plan of preparing new 8 place tables which should serve not only for occasional use, but also for constant reference and which, in point of convenience, should not need to shirk comparison with the seven place tables and at the same time were based upon the sexagesimal division of the quadrant. Discussion of the matter with eminent specialists, such as Professor H. Bruns of Leipzig, strengthened us in this intention and the project took definite shape in a memorial which was laid before the annual meeting of the Astronomical Society in Lund*). The Society approved of the plan, and promised its moral support.

The following years, 1905 and 1907, were taken up with the task of getting the means together for the work. The Royal Prussian Academy of Sciences of Berlin placed a considerable sum at our disposal as early as 1905 on the proposal of Professor Auwers, thus not only giving the undertaking a financial basis, but also lending it the weight of its scientific authority. The remaining sum necessary was not secured till the beginning of 1908 when the Imperial Academy of Sciences of Vienna on the proposal of Professor E. Weiss granted a further sum out of the Treitl bequest. With these two sums we hoped to be able to cover our expenses in connection with the preparation of the tables and in 1908 we therefore started to work out the plan. The first year was devoted exclusively to the calculations that had to be done by hand, preparatory to the process of interpolation as will be explained more fully below. The work was carried out with the aid of three or four calculators working under our constant supervision and was finished in May 1909. At the same time as the hand-calculations were begun, we entered into correspondence with Herr Hamann of Berlin-Friedenau, a constructor of calculating-machines, requesting him to put his long experience at our disposal and to help us in the construction of a new machine, by means of which the function-value was to be reckoned from the second

*) Astronomische Vierteljahrsschrift, vol. 39. 1904.

differences by summation and at once written down. M. Hamann, with a readiness for which we alwa[ys are] very grateful to him, at once acquiesced in our plan, and at the beginning of 1909 delivered us [a] machine which fully came up to our expectations. A short description of it will be given below. We immediately began work with this machine, a machine which we can highly recommend for [all] similar calculations, and with two calculators advanced so rapidly that within a year the whole enormo[us] interpolation, consisting of 828000 single values (in which we do not include a second calculati[on] of log sin and log cos) were brought to completion. Almost simultaneously with the machine work, t[he] checking of it was undertaken and the preparation of a manuscript, shortened to eight places of decim[als] for the use of the compositor.

From the very beginning, it was not our intention, nor indeed would it have been feasible with t[he] means at our disposal, to work out a completely new and independent calculation ot all the functio[n] values here presented; nor was there any pressing necessity for such a task, as in the works of the fi[rst] calculators of logarithmic tables, quoted below, a reliable foundation has once for all been provid[ed] for at least twelve decimal places. The special task we set ourselves was to arrange the calculatio[ns] we had to make so that the eighth decimal should be rendered absolutely certain and that all functio[n] values in such intervals should be given, so that not only interpolations with second differences sho[uld] be superfluous but also inconvenient interpolations with first differences, i. e., those with four fig[ure] differences should remain in a minority. For these purposes it was necessary to give the logarithms [of] all numbers from 1—200000 and the logarithms of the trigonometrical functions from second to seco[nd] and for the first degrees of the quadrant to add the well known auxiliary magnitudes S and T at su[ch] length as seemed necessary.

Such being the scope of our work, it at once became clear that the 10 place Thesaurus of Ve[ga] (Leipzig 1794) and the 10 place Vlacqs Tables*) of which it is a reprint, were unserviceable for [our] purpose; so we had to go back to the original English works, viz.

1) Briggs, Arithmetica logarithmica, Londini 1624,

2) Briggs-Gellibrand, Trigonometria britannica, Goudae 1633.

The former gives to 14 places the logarithms of the numbers from 1—20000 and from 90000—1000[00], the latter, besides other things, gives to 14 places the log sin from 36″ to 36″ (Hundredths of degre[es]). It has long been known and is confirmed by working out the differences that the 14th decimal in th[ese] possesses no reality, and might therefore have been omitted in the tables. We have therefore taken [the] figures of Briggs-Gellibrand, shortened to twelve decimals, as the basis of our calculations, and have [in] so doing reckoned the uncertainty of the twelfth decimal at a maximum of ± 0.6 units. The numer[ous] direct calculations carried out with twenty decimals which had to be done to make fully certain of [the] eighth decimal have in all cases confirmed this presupposition so that we have never had any occas[ion] to doubt the reliability of the basis on which we worked.

The carrying out of the above programme demanded the following works of interpolation:

(A) Numbers. The logarithms from 10000—20000 had to be reduced to a ten times smaller inte[rval] and then gave the logarithms of 100000—200000; the logarithms from 2000—10000 had to be redu[ced] to a ten times smaller interval and then gave the logarithms of 20000—100000. The logarithms f[rom] 90000—100000 already stood in the Arithmetica Logarithmica it is true, but for the sake of unifor[mity] have been reckoned out afresh. The above quoted Table de Service géographique de l'armée might h[ave] given us in the same way the values for 100000—120000 and from 10000—100000 to eight pla[ces] but we chose rather to reckon out these also afresh, because we wanted to guarantee the correct[ness] of all logarithms given by us in the present work by our own calculations and because we wished [to] possess a complete table with twelve decimals in the chosen interval at least in manuscript. The Fre[nch] table was used as a check in reading the proofs and only one mistake was found in it, viz., in [the] case of log 28917 where 461 15323 bas to be read instead of 461 15324. The process of interpola[tion]

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*) Vlacq, Arithmetica logarithmica Goudae, 1628 (gives to ten places the logarithms of the numbers from 1—1000[00]). Vlacq, Trigonometria artificialis Goudae, 1633 (gives to ten places the log sin, cos, tang, cotg. from 10″ to 10″).

was performed in the following way in order to discover any errors there might be in Briggs-Gellibrand. In the first place the differences up to the fourth order: $(a + \frac{1}{2}, 1)$, $(a, 2)$, $(a + \frac{1}{2}, 3)$, $(a, 4)$*) of the wide interval were reckoned out by hand to twelve places. If now in the summation by the machine only the two first series of differences are taken into account and the influence of the third and higher differences, the neglect of which might have falsified the interpolated value at most by a bare unit of the twelfth decimal, were disregarded till afterwards, the Bessel's formula would have been best adapted for the interpolation. The initial and final equations of the first series of differences in the narrower interval have here the values:

$$(a + \tfrac{1}{20}, 1) = 0.1\,(a + \tfrac{1}{2}, 1) - 0.045\,(a + \tfrac{1}{2}, 2)$$
$$(a + \tfrac{19}{20}, 1) = 0.1\,(a + \tfrac{1}{2}, 1) + 0.045\,(a + \tfrac{1}{2}, 2);$$

The second difference

$$(a + \tfrac{1}{10}, 2) \text{ up to } (a + \tfrac{9}{10}, 2) = 0.01\,(a + \tfrac{1}{2}, 2)$$

is constant for a whole interval. The only alternative, therefore, for the preparatory calculations by hand, was to exactly ascertain the initial value $(a + \frac{1}{20}, 1)$ of the first series of differences as well as the second difference for each interval. Summation with the machine then produced the interpolated logarithms, taking into account the two first differences. In order to avoid errors in rounding-off, four additional decimals were added, whereby a check on the summation was obtained, in that the final value of every single interval to the sixteenth decimal place inclusive necessarily agreed exactly with the initial value of the following interval.

Even if a knowledge of the final values $(a + \frac{19}{20}, 1)$ of the first series of differences were not positively requisite, they have yet been calculated simultaneously with the initial values of the similar series of differences, so as to achieve, even before the summation, a good check on the correctness of the preparatory calculations. For if the difference between the initial value of the first series of differences in an interval and the final value of the same series of differences is taken, it results in the decisive and easily workable check formula:

$$(a + \tfrac{1}{20}, 1) - (a - \tfrac{1}{20}, 1) = 0.01\,(a, 2) - 0.0225\,(a, 4).$$

The influence of the third differences on the functional value, in Bessel's formula, is:

$$\frac{(t - \frac{1}{2})\,t\,(t - 1)}{2 \cdot 3}\,(a + \tfrac{1}{2}, 3);$$

in those cases where it might influence the eighth decimal it has been accurately calculated and taken into account. This object was facilitated by the following table:

t = phase	Coefficient of the third difference	Maximum error
0.0	0.000	± 0.600
0.1	+ 0.006	± 0.654
0.2	+ 0.008	± 0.696
0.3	+ 0.007	± 0.726
0.4	+ 0.004	± 0.744
0.5	0.000	± 0.750
0.6	— 0.004	± 0.744
0.7	— 0.007	± 0.726
0.8	— 0.008	± 0.696
0.9	— 0.006	± 0.654
1.0	0.000	± 0.600

The maximum error inserted has been set up in units of the twelfth decimal, and calculated on the supposition that the original values could not be inaccurate by more than ± 0.6 units of the twelfth decimal.

*) We adopt here and in the following the method of denomination used by Bruns in his work Grundlinien des wissenschaftlichen Rechnens.

All the logarithms which remained doubtful through abbreviation to the eighth decimal place, tak into consideration the influence of the third differences, and the possible maximum error at the place question, have been checked by the application of the series:

$$\log(x+h) = \log x + 2M\left\{\frac{h}{2x+h} + \frac{1}{3}\left(\frac{h}{2x+h}\right)^3 + \cdots\right\}.$$

No deviation from the results calculated by the machine were found, in any single case, grea than those maximum errors previously established in theory — a clear proof that the basis on which logarithms of the numbers were worked out, and, at least for the places on which further calculatio are to be based, is thoroughly accurate, and that our assumption that the initial values could not inexact by more than at the most $\pm$ 0.6 units of the twelfth decimal was justified. We deduce from above and from the previous check through four differences the correctness (within the accepted deg of exactitude) of the entire initial values and think no criticisme possible.

B. The trigonometrical functions. As an interpolation process for log sin and log tang of first five degrees similar to that used with the number logarithms could not be applied direct, the follow method was adopted. With the assistance of the fourteen place values of log sin, log cos and number logarithms, taken from the Briggs-Gellibrand tables, the familiar auxiliary values S and T fro

$$S = \log\sin - \log\text{arc}$$
$$T = \quad S \quad - \log\cos$$

were calculated from 36″ to 36″ to twelve places, interpolated, with the aid of the machine, from 1″ to in the manner prescribed for reckoning the logarithms of the trigonometrical functions from 5° to 4 and then, worked out by hand, the logarithm of the arc in seconds was added for each single second val

The twelve-place log sin and log tang thus obtained were employed to calculate the log cos of individual seconds using the equation:

$$\log\cos = \log\sin - \log\text{tang}$$

and the agreement of these values with the values to fourteen places of the same logarithms previous made by Bruhns*), and kindly placed at our disposal through the good offices of Prof. E. Becker, serv as a strict check upon the accuracy of the whole calculation, and the material on which it was base The logarithms of the cotangent were merely included in the printed manuscript as a decimal suppleme to log tang to eight places.

In the sphere 5° to 45° the logarithms of sin, cos and tang for each 36. arc second to twelve plac have been culled from Briggs-Gellibrand, log sin and log cos as they stand, and log tang by means the small (14 places) calculation

$$\log\text{tang} = \log\sin - \log\cos\,.$$

All these values were subjected, before further elaboration, to a searching check through differen to the fourth order inclusive. It was then found that the exactitude of these values is to be estima the same as in the numericals viz: $\pm$ 0.6 units of the twelfth decimal.

For the purpose of interpolation by totalling with the machine, the differences of the one seco intervals had to be calculated from the differences of the rougher 36″ interval. This was done by usi the formulas**):

$$\left(a+\frac{1}{72},1\right) = \frac{1}{36}\left(a+\frac{1}{2},1\right) - \frac{35}{2\cdot 36^2}\left(a+\frac{1}{2},2\right)$$
$$\left(a+\frac{1}{36},2\right) \text{ up to } \left(a+\frac{35}{36},2\right) = \frac{1}{36^2}\left(a+\frac{1}{2},2\right).$$

In this calculation four further decimals were added, as previously. Inaccuracies in rounding could, it is true, not be entirely avoided, but as will be shown later on, these inaccuracies which remain in rounding off, could (in spite of their accumulation in the summation process) exert but little influen

*) Bruhns, Neues Logarithmisch-Trigonometrisches Handbuch auf sieben Dezimalen, pag. IX.

**) See Enzyklopädie der math. Wissenschaften Band I, 2, pag. 814.

on the twelfth decimal. At the same time we determined the final value of the first series of differences in every interval by the formula:

$$\left(a+\frac{71}{72},1\right)=\frac{1}{36}\left(a+\frac{1}{2},1\right)+\frac{35}{2\cdot 36^2}\left(a+\frac{1}{2},2\right),$$

so as to secure a good check on this portion of the calculation:

$$\left(a+\frac{1}{72},1\right)-\left(a-\frac{1}{72},1\right)=\frac{1}{36^2}(a,2)-\frac{35}{4\cdot 36^2}(a,4).$$

The work was then resumed by the machine. If here, too, the last value of the individual interval obtained by the machine could not agree exactly with the initial value of the interval following, in consequence of the unavoidable inexactness in rounding-off, provision was made in the preparatory hand calculation, by determining the error in each case, that the necessary check upon the correctness of the summation, in fact, upon the entire interpolation process, should be obtained through the harmonising of the theoretically derived errors with those of the summation.

The cotangents of 1″ to 1″ are independent of the tangents, certainly of that of the one second interval calculated for these differences, partly on account of the check, partly because the machine yielded the figures more quickly and accurately than it would have been possible to convert by hand the tangents into the cotangents, and finally because we desired to obtain a complete machine manuscript of the twelfth place values of all four functions. The manuscript for the printer was obtained also for the cotangents as a decimal supplement of the eight place values of the tangents for 5° to 45°.

For calculating the influence of the third differences on the twelfth decimal, use was made of the values of the coefficients $\frac{(t-\frac{1}{2})\,t(t-1)}{6}$ of the third term

$$\frac{(t-\frac{1}{2})\,t(t-1)}{6}(a+\tfrac{1}{2},3).$$

as set out in the following table:

t = phase	Coefficient of the third difference	Maximum error	Summation error	Total error
0 : 36	0.00000	± 0.6000	0.0000	± 0.6000
1 : 36	+ 0.00213	0.6162	± 0.0097	0.6259
2 : 36	+ 0.00389	0.6315	0.0188	0.6503
3 : 36	+ 0.00530	0.6459	0.0274	0.6733
4 : 36	+ 0.00640	0.6593	0.0354	0.6947
5 : 36	+ 0.00720	0.6718	0.0429	0.7147
6 : 36	+ 0.00772	0.6834	0.0498	0.7332
7 : 36	+ 0.00798	0.6940	0.0562	0.7502
8 : 36	+ 0.00800	0.7037	0.0620	0.7657
9 : 36	+ 0.00781	0.7125	0.0673	0.7798
10 : 36	+ 0.00743	0.7204	0.0720	0.7924
11 : 36	+ 0.00688	0.7273	0.0762	0.8035
12 : 36	+ 0.00617	0.7333	0.0798	0.8131
13 : 36	+ 0.00534	0.7384	0.0829	0.8213
14 : 36	+ 0.00440	0.7426	0.0854	0.8280
15 : 36	+ 0.00338	0.7458	0.0874	0.8332
16 : 36	+ 0.00229	0.7481	0.0888	0.8369
17 : 36	+ 0.00115	0.7495	0.0897	0.8392
18 : 36	0.00000	0.7500	0.0900	0.8400
19 : 36	− 0.00115	0.7495	0.0898	0.8393
20 : 36	− 0.00229	0.7481	0.0890	0.8371
21 : 36	− 0.00338	0.7458	0.0877	0.8335
22 : 36	− 0.00440	0.7426	0.0858	0.8284
23 : 36	− 0.00534	0.7384	0.0834	0.8218
24 : 36	− 0.00617	± 0.7333	± 0.0804	± 0.8137

t = phase	Coefficient of the third difference	Maximum error	Summation error	Total error
25 : 36	— 0.00688	± 0.7273	± 0.0769	± 0.8042
26 : 36	— 0.00743	0.7204	0.0728	0.7932
27 : 36	— 0.00781	0.7125	0.0682	0.7807
28 : 36	— 0.00800	0.7037	0.0630	0.7667
29 : 36	— 0.00798	0.6940	0.0573	0.7513
30 : 36	— 0.00772	0.6834	0.0510	0.7344
31 : 36	— 0.00720	0.6718	0.0442	0.7160
32 : 36	— 0.00640	0.6593	0.0368	0.6961
33 : 36	— 0.00530	0.6459	0.0289	0.6748
34 : 36	— 0.00389	0.6315	0.0204	0.6519
35 : 36	— 0.00213	0.6162	0.0114	0.6276
36 : 36	0.00000	± 0.6000	± 0.0018	± 0.6018

The quantities shown in the above table and designated maximum, summation and total errors indicated in units of the twelfth decimal. The maximum error shown has the same meaning as previo with the numeral logarithms, that is, the uncertainty of the interpolated values in consequence of assumed inaccuracy of the initial values of ± 0.6 units of the twelfth decimal. As the interpolat could not be carried through free from error in rounding-off, a further error arose, caused by the summat of values of the initial differences of the first and second order in the one second interval, such values be inaccurate by certain amounts of the 16th decimal. The total error, equal to the sum of the first two nam represents the biggest inaccuracy possible in the interpolated logarithms for every phase of the interpolati

If, after taking into consideration the influence of the third differences at the twelfth decimal occurred that, in consequence of the uncertainty of the value as regarded the total error, the abbreviat at the eighth decimal remained doubtful, these values were re-calculated from the commencement. doing this it appeared most expedient, in the first place, to ascertain the numerical values of the gonometrical functions according to the familiar series:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \mp \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \mp \cdots$$

and to determine the logarithms to these in the same way as with the logarithms of the numb Doubtful logarithms of the tangents were safeguarded by:

$$\log \text{tang} = \log \sin - \log \cos$$

after the two logarithms, to twenty places, were reckoned on to the numerical values of sinus and cos

These re-calculations of the logarithms of the trigonometrical functions, as well as the logari of the numbers, were worked out twice independently of each other, and by different methods: firs one of the authors in the manner indicated; they were then calculated a second time by Dr. Wit whom we owe our most grateful thanks. For the trigonometrical functions he was aided by a self- table of the twenty-one place logarithms of sinus and cosinus, a report on which will be made The two calculations agreed perfectly to, at least, 16 places.

No unexpected differentiations from the values obtained by the machine showed themselves in the re-c lations of the logarithms of the trigonometrical functions, so that we are justified in drawing the conclus which we deduced from the harmonising of the re-calculation of the number logarithms with the machine res

The material yielded by the machine places in our hands a big, twelve place table in an interval suc we have never had before; the twelfth decimal, it is true, is not safeguarded throughout, owing to the negle the third differences, but can be determined, roundly, to a unit by a comparatively small calculation. Ev this table cannot be printed it will serve many useful purposes; we have taken, therefore, considerable care suitable arrangement, so that any particular value can be looked up in a moment; we have had it properly bo placed it on shelves and handed it over to the permanent care of the Berlin Astronomical Rechen-Inst

We insert here a brief description of the calculating machine employed, in so far as is indispensable for a right understanding of the method of calculation adopted.

The machine consists of two similar machines, calculating-machines I and II, each to 16 places, and independent of each other; and the fly-press III (see frontispiece). Each of the two machines, again, consists of an intercalary- and a reckoning-adjustment. The method of working is as follows: a value in No. I intercalary-adjustment is added, by turning the crank-handle a, to that in reckoning-adjustment I, and simultaneously to the value in intercalary-adjustment II, and that, too, either in a positive or negative sense, according to whether the machine is set by the lever c for addition or subtraction. A turn of the second crank-handle b then adds the value lying in intercalary-adjustment II to that lying in counting-adjustment II, likewise in a positive or negative sense, according to whether the lever d is set for addition or subtraction. By turning crank-handle b the intercalary adjustment belonging to the fly-press, corresponding to the value, standing in the counting-adjustment of II, is simultaneously put in place. A further turn of crank-handle a either adds or subtracts again the value standing in intercalary-adjustment I to that in counting-adjustment I. At the same time with each turn of the crank-handle the fly-press participates, and each time the value standing in counting-adjustment II is printed off.

The figure gives a true copy (in $\frac{2}{3}$ of the natural size) of a slip, calculated and printed by the machine in about 5 minutes.

log tang

34	9	36

8316	0055	2725	0000
8316	0508	4312	2928
8316	0961	5882	1276
8316	1414	7434	5044
8316	1867	8969	4232
8316	2321	0486	8840
8316	2774	1986	8868
8316	3227	3469	4316
8316	3680	4934	5184
8316	4133	6382	1472
8316	4586	7812	3180
8316	5039	9225	0308
8316	5493	0620	2856
8316	5946	1998	0824
8316	6399	3358	4212
8316	6852	4701	3020
8316	7305	6026	7248
8316	7758	7334	6896
8316	8211	8625	1964
8316	8664	9898	2452
8316	9118	1153	8360
8316	9571	2391	9688
8317	0024	3612	6436
8317	0477	4815	8604
8317	0930	6001	6192
8317	1383	7169	9200
8317	1836	8320	7628
8317	2289	9454	1476
8317	2743	0570	0744
8317	3196	1668	5432
8317	3649	2749	5540
8317	4102	3813	1068
8317	4555	4859	2016
8317	5008	5887	8384
8317	5461	6899	0172
8317	5914	7892	7380
8317	6367	8869	0008

($\frac{2}{3}$ of the natural size.)

The condensation into tables of the numerical results obtained in a manner convenient for calculation and very easy for quick reference, has been, together with the safeguarding of the eighth decimal, looked upon by us as forming our chief task. A few general remarks on the arrangement we have selected may not be out of place.

The handiness of a table of large compass depends upon its size, the typographical grouping of its numerical matter, and the aids presented for rapid interpolation. As regards its size, in order to avoid unwieldily thick volumes, or more than two volumes, we had to resort to the largest possible format which the calculator could just comfortably find room for beside the calculating sheet and conveniently keep under his eye. The lexicon-octavo is certainly larger than the customary logarithms-table shape, but it has proved serviceable with other tables. A division into two volumes was unavoidable, but need scarcely present any difficulties as an arrangement can easily be affixed to the calculating table which will enable both books to be used at the same time. We have utilized the Bremiker division as a pattern for the typographical grouping, familiar to most calculators, and medieval brevier figures have been selected as type, now generally used and approved in most works of tables. Differences and proportional tablets are given as aids for the interpolation. With the size and interval selected there were no difficulties in the volume with the logarithms of the numbers in giving both in fullest detail; the three place differences are greatly in the majority there, and can be supplied with P. P. throughout; the four place ones only occur on 45 pages, of which 17 alone did not afford sufficient space to take in all P. P., but on these half, at least, of the P. P. are given, so that they can be conveniently used. Matters are essentially different with the trigonometrical functions; here, if an immoderate expansion of the volume, or the almost inadmissible splitting up into two books, were to be avoided, the addition of P. P. had to be abandoned from the outset. We are of opi that with four place, and even with the three place differences, the use of a detailed multiplication t (Crelle, Peters, Zimmermann, Petrick), of the sliding rule, or a four place logarithm-table in card form essentially more practicable than the use of the customary small P. P. tablet, and we saw no disadvant therefore, likely to arise from its omission. On the other hand it was imperatively necessary to su the calculator with a possible means of conveniently and accurately forming the differences. As t presentment was impossible from the start, we were obliged to have recourse to the arrangement cho in which the successive function values are placed one under the other, and the formation of the differe is best facilitated. At the head and foot of the page we have in addition been able to give from the occasional initial and final differences of the respective columns, so that by glancing at the two last dec places of the function values, the calculator obtains the difference accurately, and is safeguarded from blunders. For the small angles to $1^{\circ}12'$ the differences are in five places, and to about $2^{\circ}0'$ the sec differences are perceptible; here, therefore, a convenient difference formation and interpolation in selected interval of $1''$, below which we could hardly go, was impossible. We abandoned the atte to surmount this difficulty, and are of opinion that, in eight place calculations, the formation of trigonometrical functions of small angles to, at least, $2^{\circ}0'$, and also further to $5^{\circ}0'$ is best achieved means of the corollary quantities S and T. It is for this reason that we give these quantities minuteness nowhere previously attempted, and arranged in a way which is equally practicable for tasks of getting the function and finding the angle. We give them doubly up to $2^{\circ}46'40''$, in volumes, so that, up to the limit mentioned, the first volume alone permits an easy finding of the gonometrical functions sinus and tangens; by including sinus and tangens in the first volume the rever of the task has been rendered easily possible. From there on, in order to find a function or an a both volumes certainly are required, yet we regard this as a lesser evil than an involved interpola From $5^{\circ}0'$ on only four place differences occur up to $12^{\circ}27'$, and afterwards three place ones in entire remaining portion of the table. These may be accurately formed by glancing at the head the foot of the column, and at the last decimal of the successive functional values, so that the u of the table is no more inconvenient than that of a seven place table with a $10''$ interval, if the mentioned are set ready.

Consideration for the space at disposal in all tables of any bulk, renders a splitting up of characteristics of the logarithms, and the first decimal places, an imperative necessity. This cause difficulty in the first volume as we were aided by the arrangement familiar to the calculator, of the s place tables, namely the splitting up of the three first decimals, and the asterisks at the third deci on the other hand the method chosen in the second volume to indicate the places at which the split figure alters one unit, was a matter of much consideration. We finally selected an asterisk so pl that it cannot possibly escape the eye in glancing over the column. All the number groups abov asterisk are to be connected with the split figure group in prominent bold type at the head of the p all the number groups below the asterisk with that at the foot of the page, and this holds good wh the argument is situated above and on the left, or below and on the right. No doubt there e other ways of surmounting the difficulty, but they were not consistent with the principle we had de upon at the outset, which doubtless all calculators will approve, viz., that the four functions sin, cotg, cos in this sequence should stand immediately next one another as they are mostly used conjo The care taken that the asterisk should be clearly visible throughout the whole column hindered us from taking in hand further divisions, by heavier lines, of the 61 figure groups from about 20 t or at Second 30, which otherwise would have been very desirable.

The division of the eight decimal places into two groups which must be undertaken by the calcul as he cannot retain eight figures in his head at the same time, has been consistently so carried through the two volumes that three figures belong to the first group, and five to the second. Altho at first sight, the dividing into two equal groups appears more natural, the distribution we chosen is preferable as every experiment will prove. The 3—5 division is a practical one, too writing down.

In order to obtain a correct reproduction of the tables we adopted the following measures: Of the slips printed by the machine, and carefully checked for the third differences, each of which contained 50 values for the numbers and 36 for the trigonometrical functions, to 16 decimals, one sheet intended specially for the compositor was rounded off to 8 decimal places and set out in the way it was to appear in print. A first proof was read partly from this manuscript, partly from the original calculation, by Herr Kreuter, and thenceforward entirely kept apart from the manuscript. The corrected sheets were then compared number for number with the original calculations by Dr. Neugebauer. The copy was now stereotyped, and the stereotyped-proofs subjected to a final, most scrupulous reading which work was shared by the authors and Dr. Paetsch; all headings and accessory figures were once more revised, the figure matter tested by strict difference formations, and the eighth decimal once again compared with the original calculation. It is worthy of mention that in the final proof-reading exceptionally few mistakes were found (one in every 80 pages on the average) from which circumstance the care bestowed on the first reading is very evident. In fact the tables are as accurate as man's work can make them. Experienced men of science alone had any share in the corrections, who were fully sensible of the responsibility of their work, so that the calculator may approach the tables with the greatest confidence.

In the course of the proof-reading many comparisons were made with other tables as for instance, Bruhns, the Thesaurus of Vega, the French eight place table etc. In doing so the following mistakes were found in them:

In the Table de Service géographique de l'armée the one already indicated above.

In Bruhns seven place table

log tang $3^{\circ}3'59''$ read 8.7289195 instead of $8.728919\overline{5}$
cotg 3 3 59 » $1.271080\overline{5}$ » » 8.2710805.

In Bremiker's six place table:

log tang $3^{\circ}3'59''$ read 8.728920 instead of 8.728919.

In the Thesaurus of Vega:

log cotg $9^{\circ}\ 5'\ 50''$ read 10.7955426908 instead of 10.7955427008
log sin 12 33 40 » 9.3374209182 » » 9.3374109182.

In Callet, Tables portatives de logarithmes:

log 965 read 2.98452 73133 43792 56538
instead of 58538
log 1022 read 3.00945 08957 98693 92700
instead of 90700.

It may be well here to group together the errors discovered in the authorities. The amended figures are printed boldly.

Briggs, Arithmetica logarithmica:

log	80	read	1.90308 99869 9194	
»	1239	»	3.09307 13063 7606	(the diff. are also incorrect)
»	2534	»	3.40380 66105 4742	
»	2547	»	3.40602 89449 6363	
»	2853	»	3.45530 17716 5708	
»	2865	»	3.45712 46263 0340	
»	6957	»	3.84242 20033 5765	
»	7559	»	3.87846 43453 4146	
»	7639	»	3.88303 65100 2767	
»	8006	»	3.90341 55857 6908	
»	8007	»	3.90346 98285 0717	
»	8008	»	3.90352 40644 7126	
»	8009	»	3.90357 82936 6305	

log 8077	read	3.90725 00828 8133
» 9972	»	3.99878 22698 3173
» 9973	»	3.99882 58190 4028
» 10033	»	4.00143 08122 4640
» 10061	»	4.00264 11490 0004
» 11081	»	4. . . .
» 11440	»	4.05842 60244 5700
» 12734	»	4.10496 48455 2783
» 14486	»	4.16094 84808 6470
» 14786	»	4.16985 07018 6149
» 15251	»	4.18329 83210 7581
» 16594	»	4.21995 10857 5532
» 17254	»	4.23688 97937 0186
» 17351	»	4.23932 45097 8780
» 17509	»	4.24326 13427 2058
» 17621	»	4.24603 05511 9059
» 17941	»	4.25384 66461 9851

Gellibrand, Trigonometria britannica

log sin 1°.00	read	8.24185 53184 2289
log cos 1.79	»	9.99978 80244 4918
log cos 2.90	»	9.99944 34674 5598
tang 19.29	»	0.34999 90945
log sin 25.66	»	9.63651 78932 5594
log sin 29.92	»	9.69791 80108 3454
log cos 25.99	»	9.95369 71482 4867

Use of the tables.

The usual instructions for using the logarithm tables may, in this case, be dispensed with, a assume that no one unacquainted with logarithms is likely to select an eight place table. A few rem however, with regard to the special arrangement of our table, and the most practical and time-s method of using it may not be out of place.

The **first volume** is arranged similarly to the seven place tables with the sole exception th order to minimise the large four place differences, the logarithms of 100 000—200 000 are given in of those of 10 000—20 000. The differences are obtained by glancing at the d column — it a contains the difference of the last logarithm in the same line to the first in the line following – on the last decimal; the P. P. are given complete, with the exception of pages 204—220, though contain, at least, the half of them. In the four place differences, despite the presence of the P. I use of a multiplication table, or a four place logarithm table, is to be recommended. In the lower of each page, on the left, are to be found first from 10 to 10, then from 100 to 100 progress the conversion of the respective number of seconds into degrees, minutes and seconds, and inve as these particulars are of considerable value in astronomical and geodetical calculations — for ins in solving the Kepler equation. Next these stand, always in a tenfold smaller interval, the same versions, the pertinent auxiliary values S and T and the log sin and log tang of the proximate a It is known that

$$\log \sin x = \log x'' + S$$
$$\log \text{tang}\, x = \log x'' + T$$

and correspondingly

$$\log x'' = \log \sin x - S$$
$$\log x'' = \log \text{tang}\, x - T.$$

Log sin and log tang are added so as, when calculating small angles from log sin and log tan, to serve as argument for the extraction of S and T. The supplementing of these particulars, desirable at the start, is furnished by the table on p. 364. The seconds' conversion of larger angles than $27^\circ 46' 40''$ is facilitated by the table on p. 367. As the log cos, also, of the corresponding angle is given through the direct obtained difference S—T (to be sure with great uncertainty, which may amount to a unit of the eighth decimal) these particulars fully supplement the trigonometrical table. If a more convenient interpolation of the S and T of $2000'' = 0^\circ 33' 20''$ than the knowledge of $10''$ to $10''$ permits, is required, then the second volume may be resorted to in which the S and T for every second are specified.

The **second volume** supplies the logarithms of the trigonometrical functions, arranged in a way which scarcely calls for any further elucidation. If the degrees and minutes are above, the seconds will be found to the left, and if they are below then the seconds must be looked for on the right. The separated figures stand above and below, and remain equal for all values in the columns printed without a star. Otherwise, the functional values above the star must be joined to the separated figures standing above, and those below the star with those below. For the small angles to $5^\circ 0'$ the S and T, elucidated above, are appended, and their use is recommended at least for all angles to $2^\circ 0'$, up to which in direct calculation the second differences would need to be taken into consideration, and yet still further to 5° as the formation of the difference and the interpolation cannot be consummated so rapidly as the quest for the number logarithm in the first volume. From 3° on, above and below, are found the differences of the functional values in the respective column, so that a glance at these and the final decimals of the functional values suffices to obtain the correct difference with certainty. Proportional tablets are not given anywhere, but the use of a multiplication table or a four place logarithm table is recommended, as these without doubt accelerate the work with four place differences, and, to say the least, are not more inconvenient with the three place differences.

In both volumes it is taken for granted that the calculator will divide the eight decimals into two groups, three and five.